

Sivers Effect and Transverse Single Spin Asymmetries (SSA) in Drell-Yan Processes

In collaboration with: M. Anselmino (Torino), F. Murgia (Cagliari)

Outline:

- SSA in Drell-Yan processes: formalism and Sivers contribution;
- Other mechanisms for SSA in Drell-Yan processes;
- A simple analytical model and numerical estimates for RHIC;
- Conclusions and outlook;

SSA in D-Y processes: formalism and Sivers contribution

We consider the differential cross section in the variables:

$$M^2 = (p_a + p_b)^2 \equiv q^2 \quad y = \frac{1}{2} \ln \frac{q_0 + q_L}{q_0 - q_L} \quad [x_F = \frac{2q_L}{\sqrt{s}}] \quad \mathbf{q}_T \quad [q_T^2 \ll M^2]$$

We do not look at the angular distribution of the lepton pair production plane, which is integrated over.

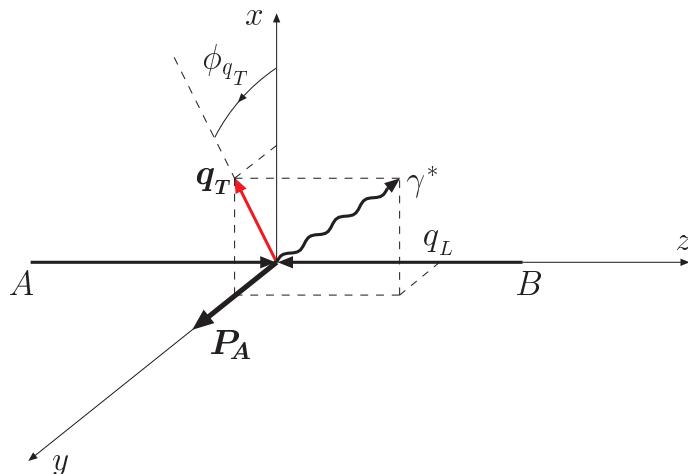


Figure 1: Our kinematical configuration. The γ^* four-momentum defines all our observables.

$$\frac{d^4\sigma^\uparrow}{dy dM^2 d^2\mathbf{q}_T} - \frac{d^4\sigma^\downarrow}{dy dM^2 d^2\mathbf{q}_T} = \frac{1}{2} \sum_{ab} \int [dx_a d^2\mathbf{k}_{\perp a} dx_b d^2\mathbf{k}_{\perp b}] \\ \Delta^N f_{a/A^\uparrow}(x_a, \mathbf{k}_{\perp a}) \hat{f}_{b/B}(x_b, \mathbf{k}_{\perp b}) \delta^4(p_a + p_b - q) \hat{\sigma}_0^{ab}$$

At LO the dominating elementary interaction is $q\bar{q} \rightarrow \gamma^* \rightarrow \ell^+ \ell^-$, [$M \ll M_Z$] so that $a, b = q, \bar{q}$ with $q = u, \bar{u}, d, \bar{d}, s, \bar{s}$ and: $\hat{\sigma}_0^{q\bar{q}} = 4\pi \alpha_{\text{em}}^2 e_q^2 / (9 M^2)$.

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \\ \frac{\sum_q e_q^2 \int d^2\mathbf{k}_{\perp a} d^2\mathbf{k}_{\perp b} \delta^2(\mathbf{k}_{\perp a} + \mathbf{k}_{\perp b} - \mathbf{q}_T) \Delta^N f_{q/A^\uparrow}(x_a, \mathbf{k}_{\perp a}) \hat{f}_{\bar{q}/B}(x_b, \mathbf{k}_{\perp b})}{2 \sum_q e_q^2 \int d^2\mathbf{k}_{\perp a} d^2\mathbf{k}_{\perp b} \delta^2(\mathbf{k}_{\perp a} + \mathbf{k}_{\perp b} - \mathbf{q}_T) \hat{f}_{q/A}(x_a, \mathbf{k}_{\perp a}) \hat{f}_{\bar{q}/B}(x_b, \mathbf{k}_{\perp b})} \\ q_T^2 \ll M^2 \implies x_a = M/\sqrt{s} e^y \quad x_b = M/\sqrt{s} e^{-y}$$

Other mechanisms for SSA in Drell-Yan processes

- $\sum_q h_{1q}(x_a, \mathbf{k}_{\perp a}) \otimes \Delta^N f_{\bar{q}\uparrow/B}(x_b, \mathbf{k}_{\perp b}) \otimes d\Delta\hat{\sigma}^{q\bar{q} \rightarrow \ell^+ \ell^-}$ [Boer 99]

h_{1q} : transversity of quark q (inside hadron A)

$$d\Delta\hat{\sigma} = d\hat{\sigma}^{\uparrow\uparrow} - d\hat{\sigma}^{\uparrow\downarrow} \simeq \cos 2\phi \quad [\phi = \widehat{P_A N}_{\ell^+ \ell^-}] \quad \int d\phi \Rightarrow 0$$

- Contributions from higher twist quark-gluon correlation functions [Qiu-Sterman (91), Boer et al. (97-02), Hammon et al. (97)] vanish upon integration over ϕ .
- The model based on orbital angular momentum and surface effects [Boros et al. (93)] might be related to the Sivers effect.
- Brodsky et al. (02) provide an important pedagogical example of SSA in $\gamma^* p^\uparrow \rightarrow q (q\bar{q})_0$ and $\bar{q} p^\uparrow \rightarrow \gamma^* (q\bar{q})_0$ where $(q\bar{q})_0$ is a spectator scalar diquark. The SSA obtained in these two cases turn out to be opposite, as predicted by Collins (02).

A simple analytical model and numerical estimates

Using [See Sivers function parameterization]:

$$\begin{aligned}
 \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) &= \Delta^N f_{q/p^\uparrow}(x) [2 e (\alpha^2 - \beta^2)]^{1/2} \beta^2 / \pi (\mathbf{k}_\perp)_x e^{-\alpha^2 k_\perp^2} \\
 \Delta^N f_{q/p^\uparrow}(x) &= \mathcal{N}_q(x) 2 f_{q/p}(x) \quad \hat{f}_{q/p}(x, \mathbf{k}_\perp) = f_{q/p}(x) \beta^2 / \pi e^{-\beta^2 k_\perp^2} \\
 \Rightarrow A_N(M, y, \mathbf{q}_T) &= \bar{\beta}^2 \frac{\beta^2 + \bar{\beta}^2}{(\alpha^2 + \bar{\beta}^2)^2} [2 e (\alpha^2 - \beta^2)]^{1/2} (\mathbf{q}_T)_x \\
 &\times \exp \left[- \left(\frac{\alpha^2}{\alpha^2 + \bar{\beta}^2} - \frac{\beta^2}{\beta^2 + \bar{\beta}^2} \right) \bar{\beta}^2 q_T^2 \right] \frac{1}{2} \frac{\sum_q e_q^2 \Delta^N f_{q/p^\uparrow}(x_a) f_{\bar{q}/p}(x_b)}{\sum_q e_q^2 f_{q/p^\uparrow}(x_a) f_{\bar{q}/p}(x_b)}
 \end{aligned}$$

with $\beta = \beta(x_a)$, $\bar{\beta} = \beta(x_b)$ and $\alpha^2 = \beta^2/r$.

Notice: this results comes from neglecting terms of the order k_\perp^2/M^2 .

With an x and flavour-independent β factor (from unpol. cross sections), the dependences of the SSA on \mathbf{q}_T and x_a, x_b (or M, y) are completely uncorrelated:

$$\begin{aligned} A_N(M, y, \mathbf{q}_T) &= \mathcal{Q}(q_T, \phi_{q_T}) \mathcal{A}(M, y) \\ &= 2 \frac{[2 e r^3 (1 - r)]^{1/2}}{(1 + r)^2} \beta q_T \cos \phi_{q_T} \exp \left[-\frac{1}{2} \frac{1 - r}{1 + r} \beta^2 q_T^2 \right] \\ &\times \frac{1}{2} \frac{\sum_q e_q^2 \Delta^N f_{q/p^\uparrow}(x_a) f_{\bar{q}/p}(x_b)}{\sum_q e_q^2 f_{q/p^\uparrow}(x_a) f_{\bar{q}/p}(x_b)} \end{aligned}$$

$\mathcal{Q}(q_T)$ has a maximum at $q_T^M = \sqrt{(1 + r)/(1 - r)}/\beta$, where its value is $\mathcal{Q}_M = [2 r/(1 + r)]^{3/2}$. For $r = 0.7$, $q_T^M \simeq 2.38/\beta$, and $\mathcal{Q}_M \simeq 0.75$.

We will adopt $\beta = 1.25 \text{ (GeV/c)}^{-1}$ (from our unpol. cross section analysis).

Apart from studying the dependence of the asymmetry upon the magnitude of \mathbf{q}_T , we consider the asymmetry averaged over \mathbf{q}_T up to a maximum magnitude q_{T1} :

$$\begin{aligned}\langle |A_N(M, y, \mathbf{q}_T)| \rangle_{q_{T1}} &= \frac{\int^{q_{T1}} d^2\mathbf{q}_T |A_N(M, y, \mathbf{q}_T)| d\sigma}{\int^{q_{T1}} d^2\mathbf{q}_T d\sigma} \\ &= Q(q_{T1}) \mathcal{A}(M, y)\end{aligned}$$

$$Q(q_{T1}) \equiv \frac{1}{\pi} r^{3/2} \left(2e \frac{1-r}{1+r} \right)^{1/2} \left\{ \sqrt{\pi} \operatorname{Erf}(w) - 2w e^{-w^2} \right\}$$

where $w = \beta q_{T1} / \sqrt{1+r}$; in particular, when $q_{T1} \rightarrow \infty$, one finds:

$$\langle |A_N(M, y, \mathbf{q}_T)| \rangle_\infty = \left(\frac{2e}{\pi} \right)^{1/2} r^{3/2} \left(\frac{1-r}{1+r} \right)^{1/2} \mathcal{A}(M, y)$$

We will consider for RHIC the following kinematical region:

$$5 \text{ GeV} < M < 10 \text{ GeV} \quad -2 < y < 2 \quad [x_{\text{Bj}} > 10^{-3}].$$

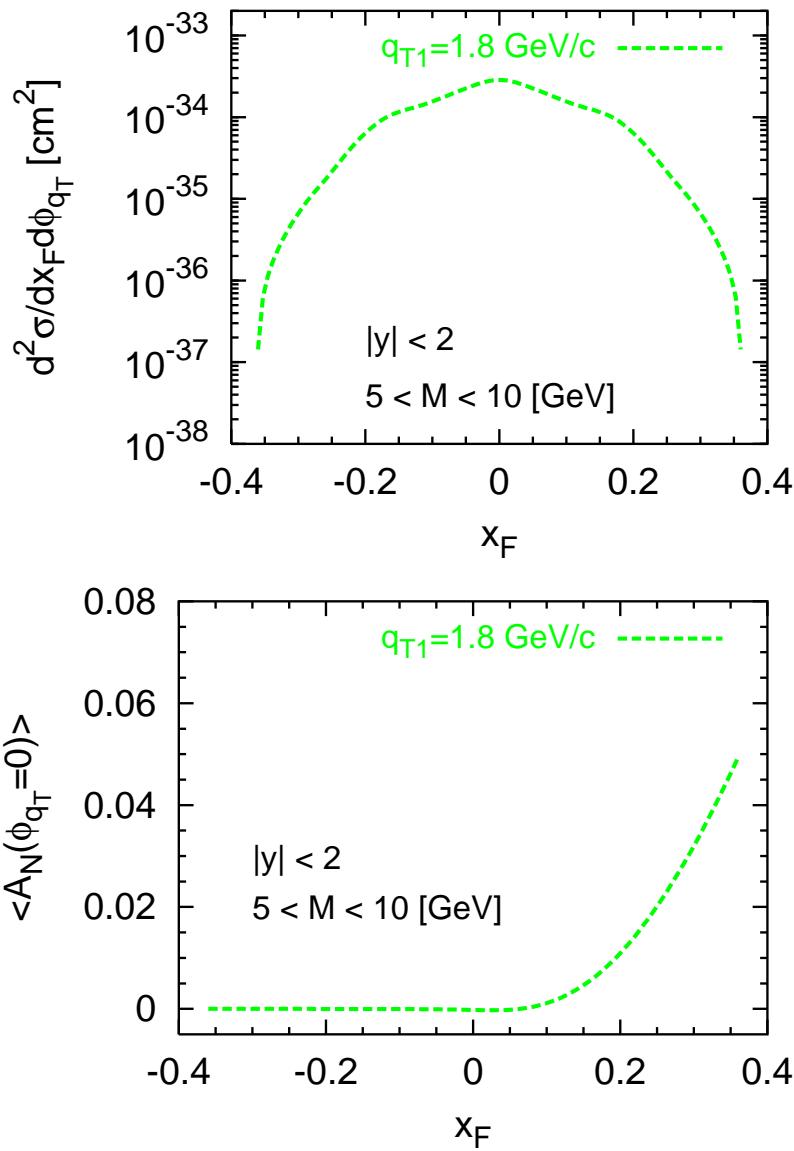
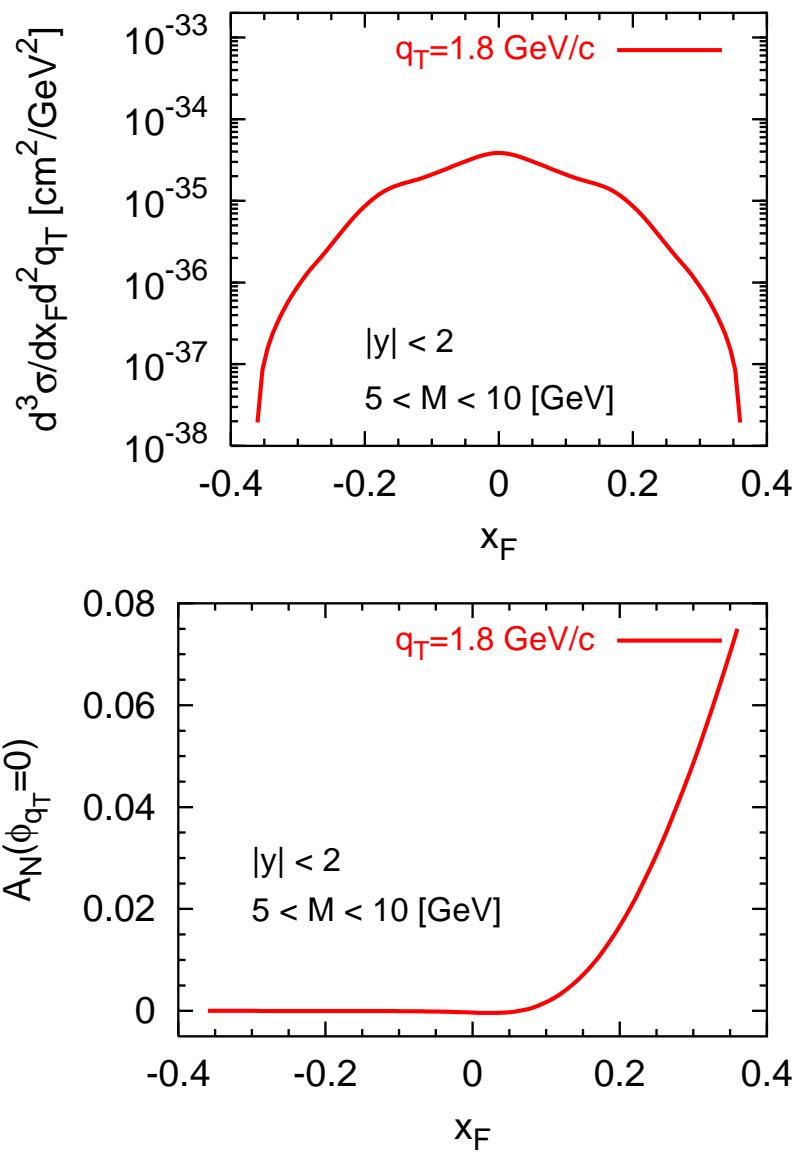
- $(M, y)[(x_a, x_b)]$ dependences:

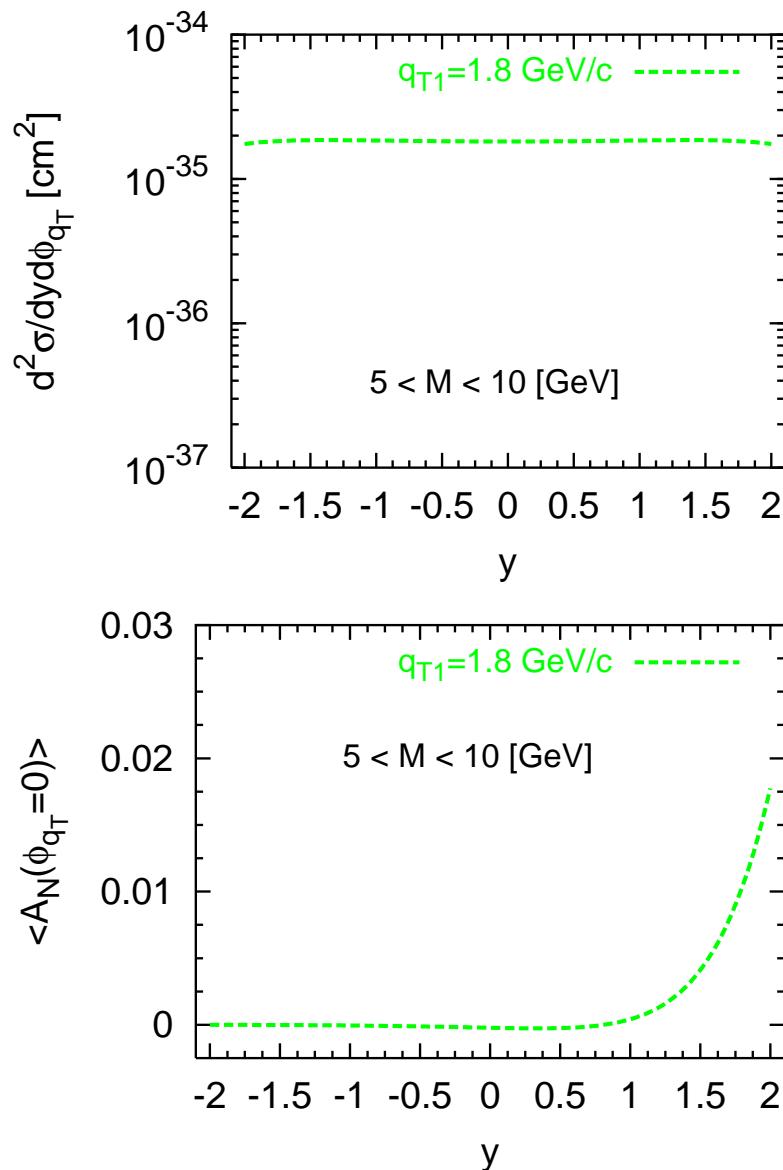
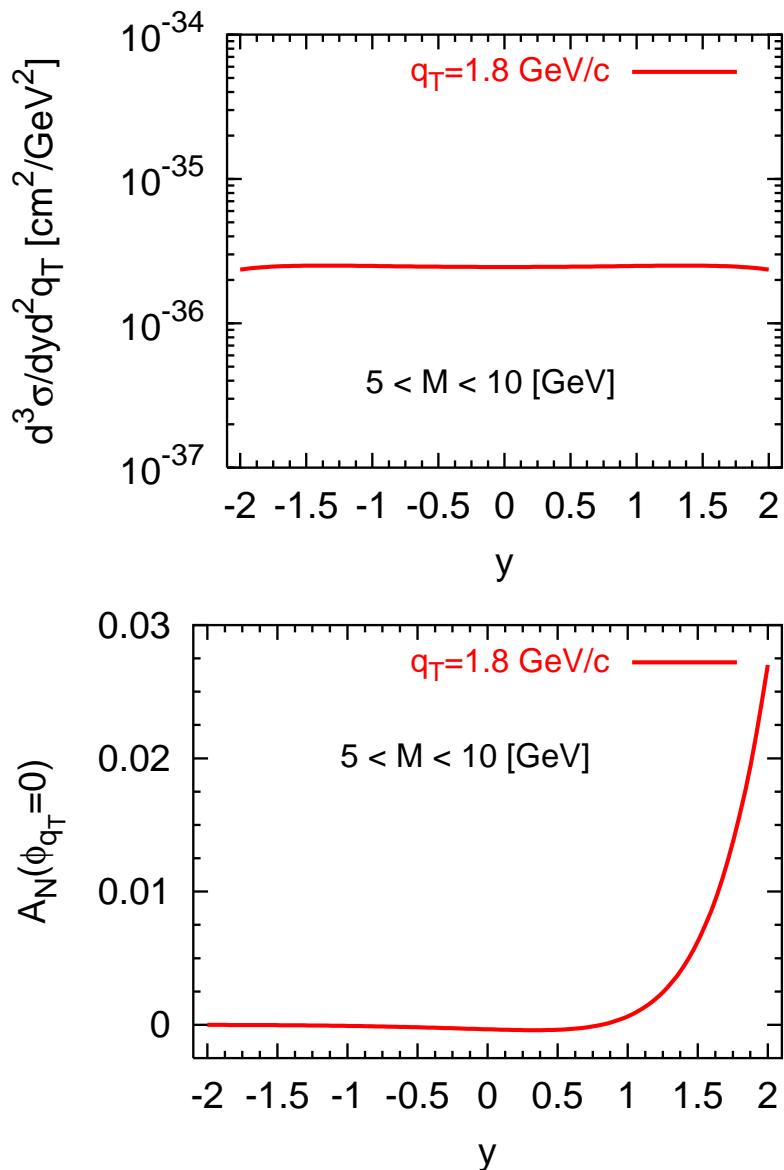
We plot asymmetries (cross sections) as functions of x_F and y averaged (integrated) over M , of M averaged (integrated) over y and of x_a averaged (integrated) over x_b .

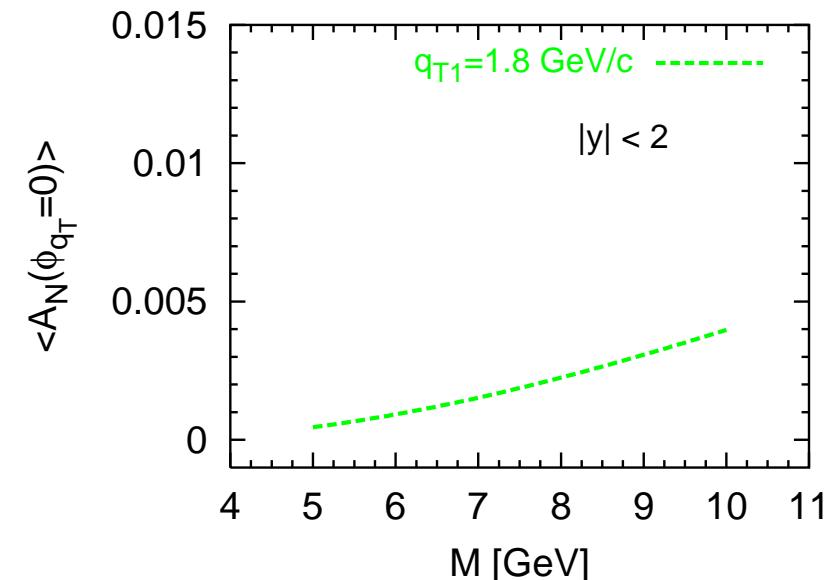
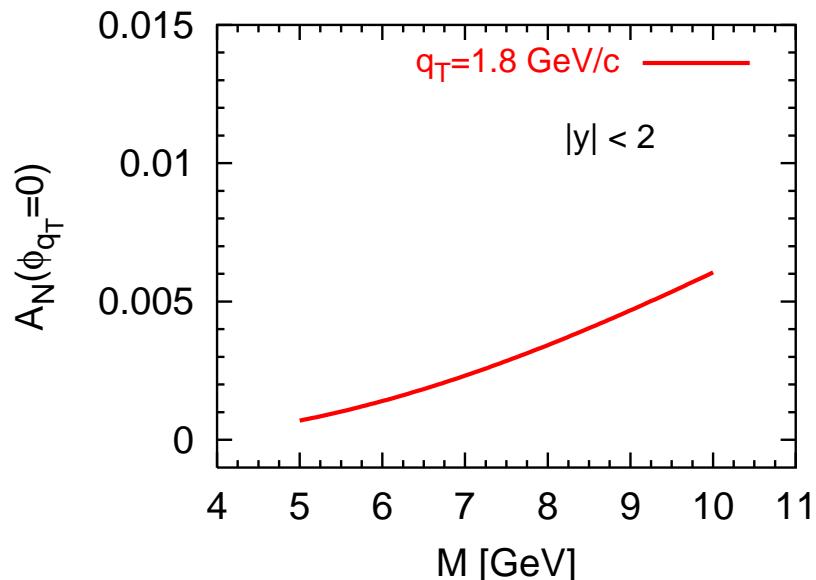
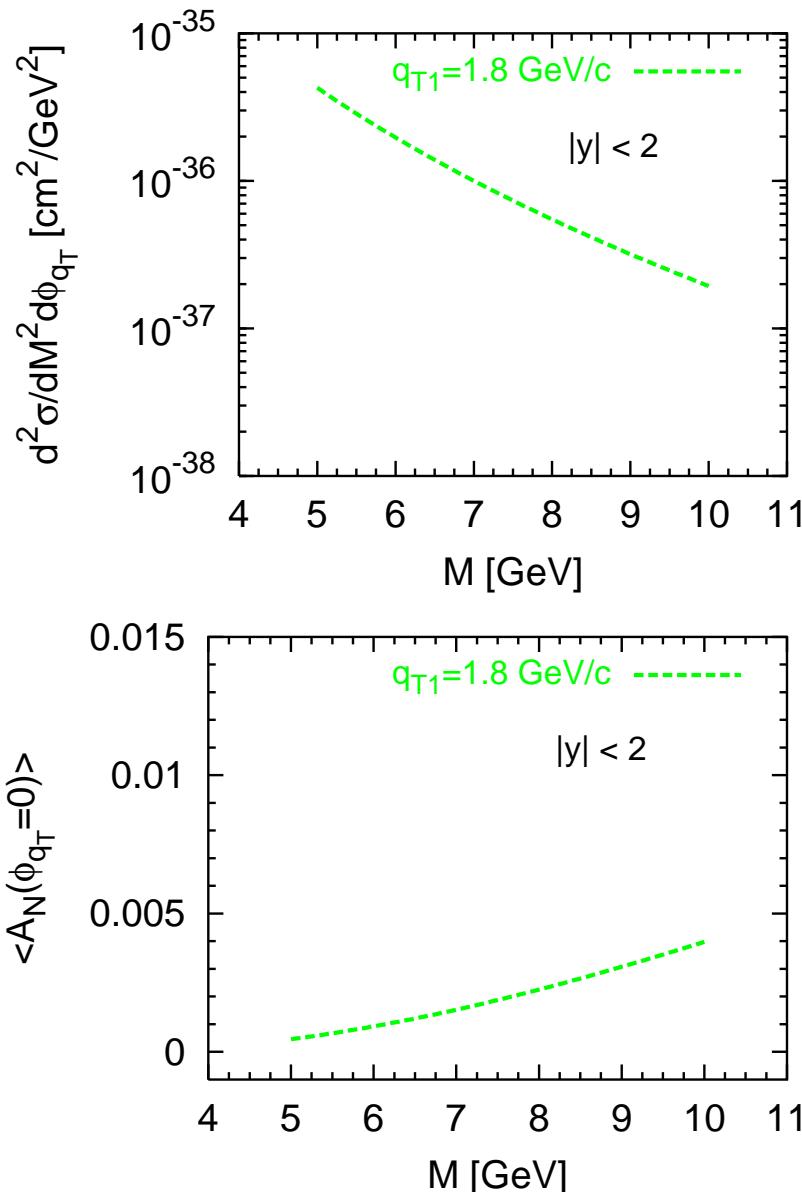
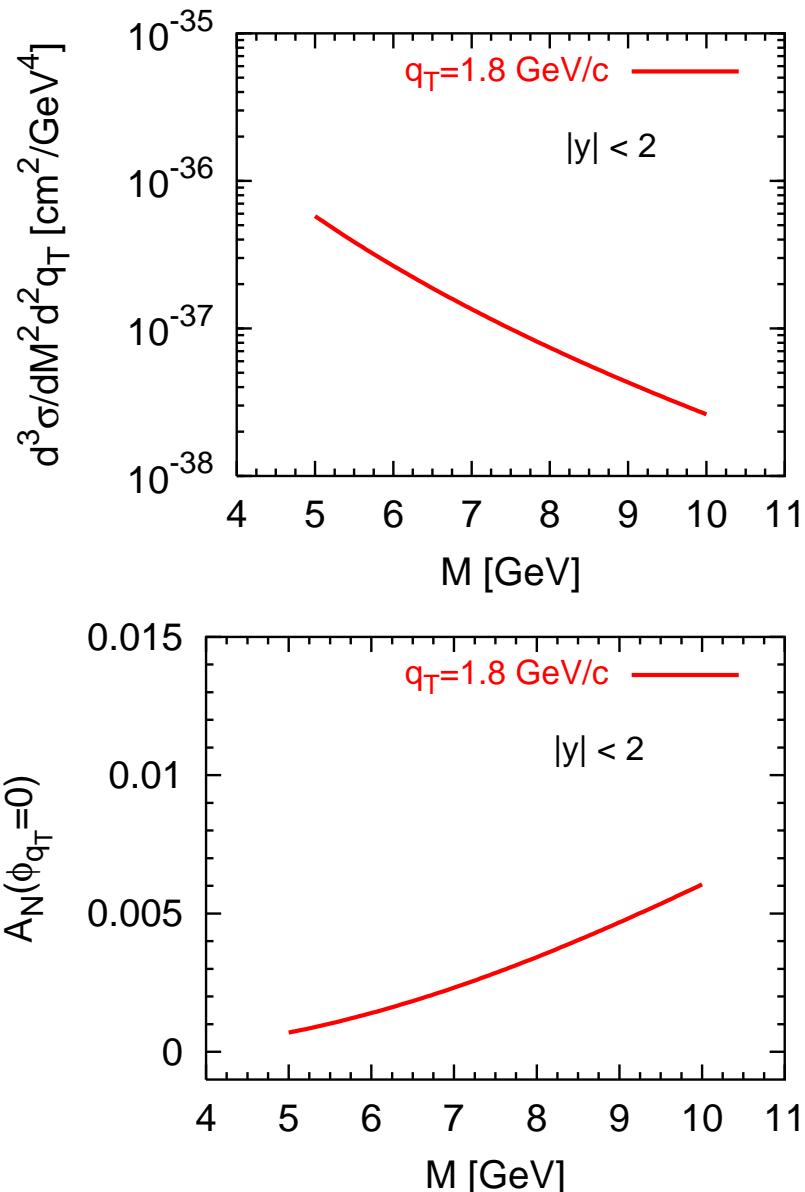
- q_T dependence:

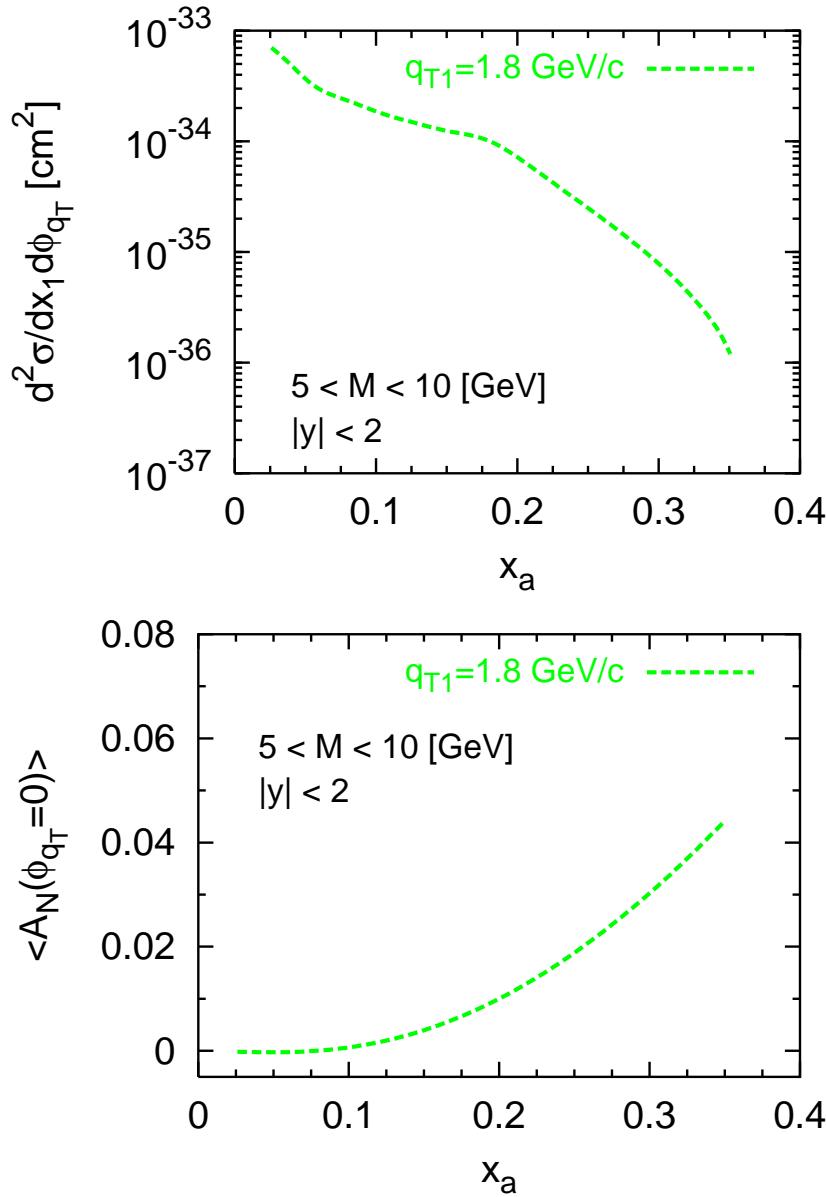
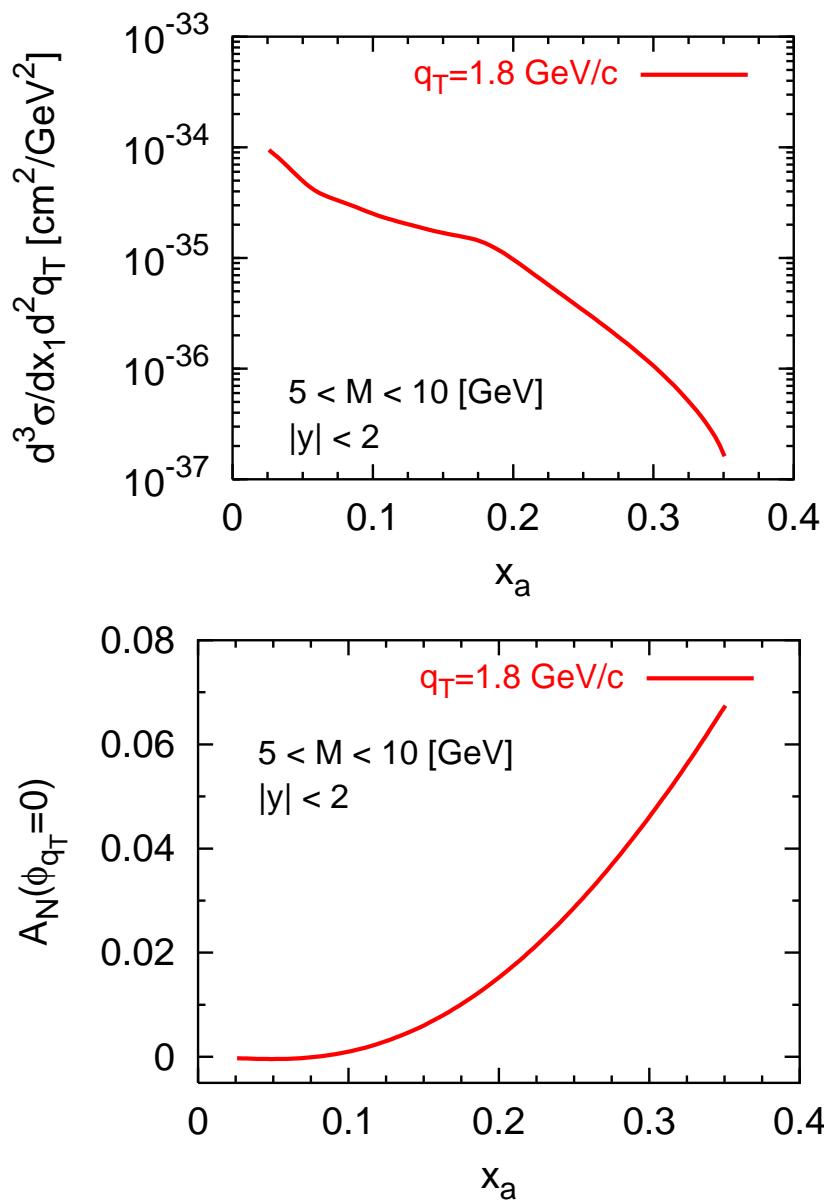
For each case (above) we show the unpolarized cross section (top-left) and the asymmetry (bottom-left) at fixed $q_T = 1.8 \text{ GeV}/c$ [$\simeq q_T^M$, A_N drops to 50% of the plotted value at $q_T \simeq 0.5 \text{ GeV}/c$] and the unpolarized cross section (top-right) and the asymmetry (bottom-right) averaged over q_T up to $q_{T1} = 1.8 \text{ GeV}/c$ [$\simeq \infty$, $|A_N|$ drops to 50% of the plotted value at $q_{T1} \simeq 0.6 \text{ GeV}/c$] at $\phi_{q_T} = 0$. Averaging $|A_N|$ also over ϕ_{q_T} in $[0, 2\pi]$ gives an extra factor $2/\pi$ (the unpol. cross section gets a trivial factor 2π).

We also show the region spanned in x_a, x_b for each case.

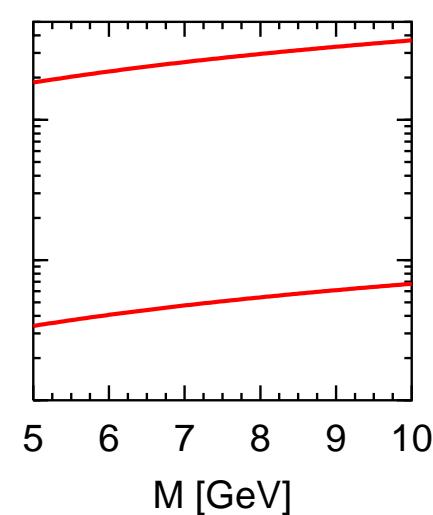
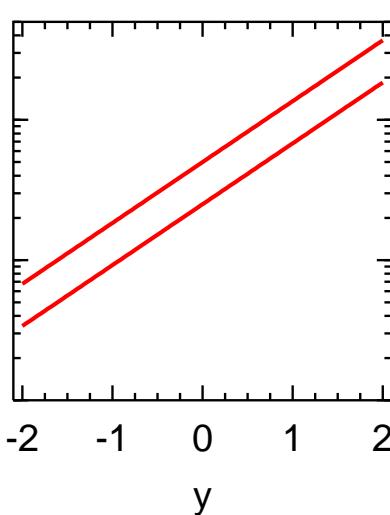
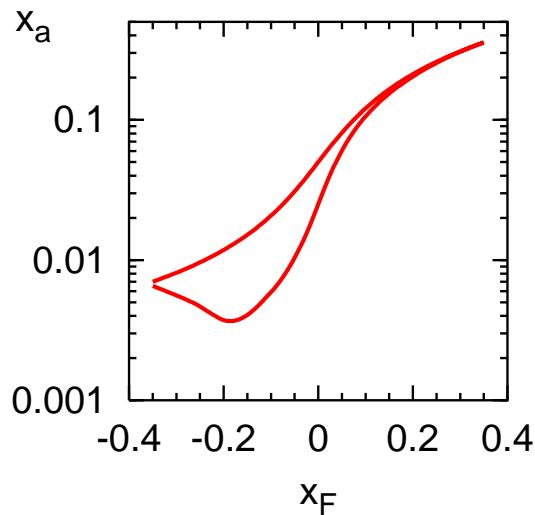
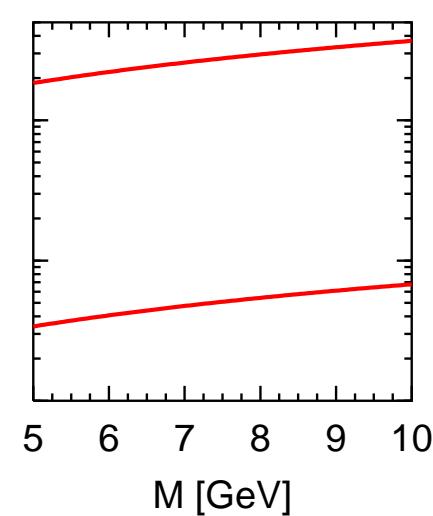
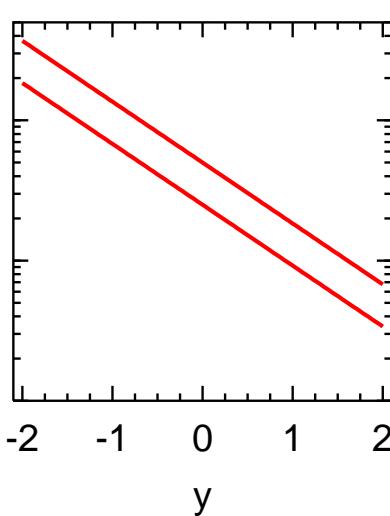
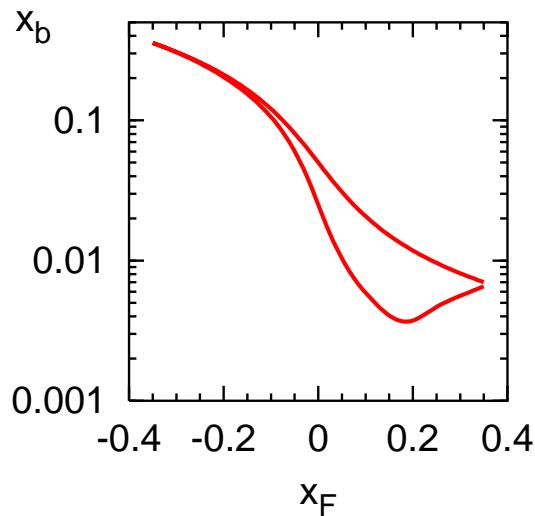








$(x_a - x_b)$ region: $5 < M < 10$ [GeV], $|y| < 2$ $\sqrt{s} = 200$ GeV



Conclusions and outlook

- Calculations of SSA originating from Sivers effect in polarized D-Y processes $p^\uparrow p \rightarrow \ell^+ \ell^- X$:
 - No Collins effect (had. fragmentation)
 - Integration over the di-lepton production plane \implies no other sources of SSA
Clean place to study Sivers effect
- general model (Gaussian): simple expression for A_N ;
- Estimates of A_N for RHIC kinematics vs. several variables (based on analysis of pion E704 data);
- small values for A_N (up to 10%) but hopefully measurable at RHIC;
- RHIC: important source of information both on the validity of our approach and on our estimates (feedback to E704 data analysis and Collins effect).